Distance regular Cayley graphs, a summer 2007 project directed by Ken W. Smith

A **graph** consists of a finite set of "vertices" with a symmetric relation ("adjacency") on the vertex set. One may construct a graph from a finite group G by identifying the vertices of the graph with the elements of G; then, given a subset S of G, we define adjacency by

$$x \sim y \iff xy^{-1} \in S.$$

(Usually we require the set S to be closed under inverses.) This graph is the **Cayley graph** $C(S, G)$.

The **distance** between two vertices x and y is the length of the shortest path between them. Given two vertices of distance k, we define $p_{ij}^k(x, y)$ to be the number of vertices z of distance i from x and distance j from y. If this parameter only depends on k, never on x and y, then the graph is **distance regular**.

Distance regular graphs of diameter d have the special property that the adjacency matrix generates an algebra of dimension $d + 1$ which is also closed under the "Hadamard" (entrywise) product. This phenomena provides a rich interplay between different areas of group theory, combinatorics and linear algebra.

Distance regular graphs of diameter two are said to be strongly regular. A **strongly regular graph** which is a Cayley graph $C(S, G)$ is equivalent to the existence of a **partial difference set** S in G.

In this project we use group theory to construct interesting distance regular graphs of small diameters (primarily diameters 3 or 4) by requiring that the graph also be a Cayley graph. Along with exploring the interplay of group theory and linear algebra, we will also do some elementary programming using the software *GAP*.