NSF - REU 2007

Project 1:

MINIMUM SEMIDEFINITE RANK OF A GRAPH

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A *graph* G consists of a nonempty set of n vertices (usually $\{1, \cdots, n\}$) and a set of edges (an edge is a two element subset of vertices). What we call a graph is sometimes called a simple, finite, undirected graph.

The set of n-by-n positive semidefinete matrices over $\mathbb{R}$ or $\mathbb{C}$ will be denoted by $\mathcal{P}_n(\mathbb{R})$ or $\mathcal{P}_n(\mathbb{C})$. For $A \in \mathcal{P}_n$, the *graph of* A, denoted $G(A)$ is the graph with vertices $\{1, \cdots, n\}$ and edges $\{\{i, j\} : i \neq j \text{ and } a_{ij} \neq 0\}$. The graph $G(A)$ is independent of the diagonal entries of A.

The set of positive semidefinite matrices of graph G over $\mathbb{R}$ or $\mathbb{C}$ is defined to be

$$\mathcal{P}(G, \mathbb{R}) = \{A \in \mathcal{P}_n(\mathbb{R}) : G(A) = G\}$$

and

$$\mathcal{P}(G, \mathbb{R}) = \{A \in \mathcal{P}_n(\mathbb{C}) : G(A) = G\}$$

The *minimum semidefinite rank* of a graph $\mathbb{G}$ over $\mathbb{R}$ or $\mathbb{C}$ is defined to be

$$mr_+(G) = min\{rankA : A \in \mathcal{P}(G, \mathbb{R})\}$$

and

$$msr(G) = min\{rankA : A \in \mathcal{P}(G, \mathbb{C})\}$$

It is clear that $msr(G) \leq mr_+(G)$ but it is not known if strict inequality holds for some G.

In this project we would like to develop a catalog of $mr_+(G)$ and $msr(G)$ for several infinite families of graphs. For known results on the minimum semidefinite rank look at the CMU REU reports from 2003-2006.

This project requires knowledge of a first course in linear algebra.