

## Assignment 7

Due Monday, November 6, 2006

This assignment is worth 30 points.

1. Every Sudoku puzzle solution is a  $9 \times 9$  matrix  $A$  which uniquely describes an ordered set  $S = \{\sigma_1, \sigma_2, \dots, \sigma_9\}$  of nine permutations such that  $A = \sum_{i=1}^9 iP_{\sigma_i}$ , where  $P_{\sigma}$  is the permutation matrix associated with  $\sigma$ .

Solve the Sudoku puzzle at <http://www.gocomics.com/foxtrot/2006/10/15/>. Then provide the ordered set  $S = \{\sigma_1, \sigma_2, \dots, \sigma_9\}$  which describes that solution.

2. (Hoffman & Kunze, page 198, problems 3 & 4.) Let

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ -2 & -2 & 2 & 1 \\ 1 & 1 & -1 & 0 \end{pmatrix}.$$

Show that the characteristic polynomial of  $A$  is  $x^2(x-1)^2$  and that this is also the minimal polynomial of  $A$ .

Is  $A$  similar (over  $\mathbb{C}$ ) to a diagonal matrix? Why/why not?

3. (Hoffman & Kunze, page 198, problem 5.) Suppose  $T$  is an operator on an  $n$ -dimensional vector space and suppose that some positive integer power of  $T$  is zero. Let  $k$  be the smallest positive integer such that  $T^k = 0$ . Show that  $k \leq n$ .
4. Suppose  $T$  is a nonzero linear operator which has the property that  $T^2 = 0$ .
  - (a) Show that for any nonzero vector  $\alpha$  either  $T\alpha = 0$  or the set  $\{\alpha, T\alpha\}$  is an independent set.
  - (b) Find, up to similarity, all nonzero  $2 \times 2$  matrices  $A$  such that  $A^2 = 0$ .
  - (c) Suppose  $A$  is a  $3 \times 3$  matrix with minimal polynomial  $T^2$ . Find, up to similarity, all such matrices  $A$ . (Hint: I separated this problem into two cases, depending on the nullity of  $A$ .)