Assignment 3

This assignment is worth 15 points and is due at the beginning of class on Monday, September 25, 2006.

1. Let A be the 3×5 matrix $\begin{pmatrix} 3 & 6 & 9 & 1 & 1 \\ 5 & 10 & 15 & 1 & -1 \\ 7 & 14 & 21 & 5 & 13 \end{pmatrix}$. Note that $rref(A) = \begin{pmatrix} 1 & 2 & 3 & 0 & -1 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$.

   Let $T_A : \mathbb{R} \to \mathbb{R}$ be the linear transformation defined by $T_A(\alpha) = A\alpha$.

   Find a basis for the following subspaces

   (a) The row space of A

   (b) The column space of A

   (c) The nullspace of T_A.

   (d) The range of T_A.

2. Let V the vector space of polynomials with real coefficients. (That is, $V = \mathbb{R}[x]$.) Let D be the derivative operator, M the "multiply by x" operator ($M(f) := xf$) and S the "integration" operator ($S(f) := \int_0^x f(t)\ dt$.)

   (a) Find the kernels and ranges of D, M, S. Which functions are nonsingular? Which are invertible?

   (b) Let $B = \{1, x, x^2, x^3, \ldots\}$ be an ordered basis for V. (Note that B is countably infinite.) Represent the operators D, M, and S as (infinite) matrices with respect to this basis.

   (c) Find linear transformations f and g in $L(V, V)$ so that fg is the identity function but neither f nor g are invertible.

3. (p. 86, # 7.) Let V and W be vector space over the field F and U be an isomorphism from V onto W. Prove that $T \to UTU^{-1}$ is an isomorphism of $L(V, V)$ onto $L(W, W)$.