Assignment 1

This assignment is due at the beginning of class on **Wednesday, September 6.** Each problem is worth five points. (Please read the expectations for turn-in assignments, as listed in the class syllabus.)

1. Prove:
   (a) that each field of characteristic zero contains a copy of the field of rationals $\mathbb{Q}$.
   (b) Prove that the interchange of two rows of a matrix can be accomplished by a finite sequence of elementary row operations of the other two types.

2. Let $A = \begin{pmatrix} 1 & 4 & 1 & 1 \\ 4 & 1 & 1 & 1 \\ 1 & 4 & 4 & -2 \\ 4 & 4 & 4 & 4 \end{pmatrix}$
   (a) Assume A is a matrix over a field of characteristic zero (such as the field of rationals, $\mathbb{Q}$.) Put A in row-reduced form.
   (b) Put A in row-reduced form, under the assumption that the underlying field has characteristic 2.
   (c) Put A in row-reduced form, assuming the underlying field has characteristic 3.

3. Let $A = \begin{pmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{pmatrix}$
   (a) Find all solutions of $AX = 0$ by row-reducing A. (Find answers for *all* fields!)
   (b) Assuming that A is a matrix over a field of characteristic zero, find elementary matrices $E_1, E_2, \ldots E_n$ such that $E_1 E_2 \cdots E_n A$ is row-reduced.
   (c) For which triples (y_1, y_2, y_3) does the system of equations $AX = Y$ have a solution? (You may assume your field has characteristic zero.)