Take-home Exam, Spring 2007

Due at noon, Friday, March 2, 2007

You may collaborate with others. In particular, please set aside **5-6:15 PM** on **Feb 19, Feb 26** and **Feb 28** to discuss these problems in our regular classroom, PE 223.

Please follow the instructions for Assignments. Some of your grade will be based on your mathematical writing "style."

Each problem is 10 points.

1. Complete, as fully as possible, the (a, b, c) permutation problem as described in previous assignments.

2. List the conjugacy classes of A_6. For each conjugacy class, choose a representative element x and describe the centralizer group $C_{A_6}(x)$.

3. Find the group of symmetries of the cube (in $\mathbb{R}^3$.) Then find the number of ways to color
   (a) the faces of the cube using up to n colors.
   (b) the edges of the cube using up to n colors.

4. Prove that A_∞ is simple.

5. Describe the Sylow 2-subgroups of
   (a) S_5
   (b) $SL(2, 5)$.

Comments on the Take-Home Exam.

1. *Please* collaborate! (It is time to finish off this problem!)

2. Of course, the conjugacy classes of A_6 are given in the book. But please defend your answers.

3. There are 24 symmetries of the cube in $\mathbb{R}^3$. Can you describe them all? (You might want to have a cube in front of you.)

4. A_∞ is described in Assignment 5, problem 5. Modify (*carefully*!) the proof of the simplicity of A_n to prove that A_∞ is simple.

5. All you need to know about Sylow subgroups (for this problem) is the definition (page 78) and the fact that they exist.

   SL(n,k) is described in exercise 2.2 on page 23. Both S_5 and $SL(2, 5)$ are nonabelian groups of order 120 so the Sylow 2-subgroups must have order 8. What do you know about groups of order 8? (One more hint: Note that in $\mathbb{Z}_5$, the element 2 has order 4 under multiplication, just like the complex number i, so that the matrix $A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$ behaves like the matrix A in problem 2.24 on page 29.)