Quiz 2, Spring 2007

This quiz is worth five points.

1. Define
   (a) normal subgroup,
   (b) kernel.
2. Let $f : G \to H$ be a group homomorphism with kernel K.
   (a) Prove that K is a subgroup of G. (You may assume that $f(1_G) = 1_H$ and that $f(x^{-1}) = f(x)^{-1}$.)
   (b) Prove that K is a <u>normal</u> subgroup of G.
   (c) Let S be a subgroup of G. Prove that $f(S)$ is a subgroup of H.

Quiz 3, Spring 2007

This quiz is worth five points.

Let $f : G \to H$ be a group homomorphism with kernel K.

1. Prove that if 1_G is the identity of G then $f(1_G)$ is the identity of H.
2. Prove that if $x \in G$ then $f(x^{-1}) = f(x)^{-1}$.
3. Prove that K is a subgroup of G.
4. Prove that K is a <u>normal</u> subgroup of G.

Quiz 3.5, Spring 2007

(This quiz was going to be Quiz 4 at one time...)

1. For each of the elements $x \in A_5$, below, first <u>describe</u> the conjugacy class of x in A_5, then <u>describe</u> the centralizer subgroup $C_{A_5}(x)$. (Your description, in each case, should, in addition to other things, <u>include</u> a statement about *size*; your description of the conjugacy class should include giving the size of the class; your description of the centralizer subgroup includes giving the size of the subgroup.)
   (a) $x = (1, 2, 3)$.
   (b) $x = (1, 2)(3, 4)$.
   (c) $x = (1, 2, 3, 4, 5)$.
   (d) $x = (2, 1, 3, 4, 5)$.
2. Let G be the alternating group A_4. For each conjugacy class of A_4, pick an element x of that conjugacy class and write out the centralizer group $C_G(x)$.

   Then show that the index of the centralizer subgroup $C_G(x)$ is size of the set of conjugates of x.

   For example, if $x = 1$ then $x^G = \{1\}$ and $C_G(x) = A_4$. Clearly $12 = 1 \cdot 12$.

   Continue with $x = (1, 2, 3)$ and so on...