

Assignment 9

This assignment is due on Wednesday, April 25, at the beginning of class. Each problem is five points.

1. Prove that if G is a finite p -group and K is the center of G then K does *not* have a complement in G .
2. (Rotman 7.4, page 155.) Give an example of an extension, K by Q , which does *not* have a subgroup isomorphic to Q .
3. Construct, up to isomorphism, all groups of the form $C_{13} \rtimes C_{12}$.
4. Let K be the noncyclic group of order nine and H the cyclic group of order four. Construct three distinct (nonisomorphic) groups of the form $K \rtimes H$.
5. (Modification of Rotman 7.6, page 155.) Which of the following properties are inherited by extensions? That is, suppose both K and Q have property P , below. If G is an extension K by Q , then must G have property P ? For each property P given below, either give an argument that “meta- P implies P ” or give a counterexample.
 - (a) P is the property “is cyclic,”
 - (b) P is the property “is a p -group,” (p is a fixed prime.)
 - (c) P is the property “is abelian,”
 - (d) P is the property “is solvable,”
 - (e) P is the property “is nilpotent.”
6. For each of the groups below, find the order of the group and then find a composition series.
 - (a) $GL(2, 2) \times S_4$,
 - (b) $GL(2, 3)$,
 - (c) $GL(3, 3)$,
 - (d) $GL(3, 7)$,
 - (e) $GL(2, 9)$,
 - (f) $GL(4, 17)$.
7. In problem 1 of Assignment 7 you showed that if a group G has k Sylow p -subgroups then there is a homomorphism from G into S_k . Find the kernel of this homomorphism.
8. Let p be a prime and G a group of order pr with r Sylow p -groups. Show that G has no subgroup of order $2p$. (Note that A_4 is an example of this phenomenon; there $p = 3$ and $r = 4$.)

Suggestions, hints for this assignment

1. Suppose the center K did have a complement (call it H .) What would that force? (Note that H is also a p -group.)
2. The example should be familiar. . . .
3. The element 2 is a primitive root of 13, that is, $\langle 2 \rangle = U(13)$.

4. Think of K as a vector space of dimension two over the field with three elements. Automorphisms of K can then be viewed as 2×2 matrices. (Do you know a 2×2 matrix of order four?)
5. Use the correspondence theorem and your collection of examples of small groups.
6. Mainly computations using the ideas from chapter 8
7. This is an example of “definition tracing.”
8. Suppose H is a subgroup of G of order $2p$. Then H has its own Sylow p -subgroup S (of index two in H .) Show that this leads to a contradiction.