Assignment 3, Spring 2007
Due at the beginning of class, Monday, January 29

1. For each of the subgroups K of groups G given in problem 2 of Assignment 2, determine if coset multiplication is well-defined. (If coset multiplication is well-defined, provide a Cayley table. If it is not, provide an example to defend you claim.)

2. For each of the subgroups K of groups G given in problem 2 of Assignment 2, determine if the set of right cosets of K is the set of left cosets of K. (If the set of right cosets is equal to the set of left cosets, can you give an argument which shows this is true? If the set of left cosets is *not* equal to the set of right cosets, give an element $a \in G$ such that $Ka \neq aK$.)

3. Let $G = (\mathbb{R}, +)$ be the set of real numbers under addition. Let $K =< 2\pi >$ be the cyclic subgroup generated by 2π.
   (a) Describe, as fully as possible, the set G/K. (What is the cardinality of G/K? What is the cardinality of each of the members of G/K?)
   (b) What are the elements of $K + \pi$?
   (c) What are the elements of $< K + \frac{\pi}{4} >$?

Comments on Assignment 3.

1., 2. These are straightforward calculations intended to provide you with a set of examples regarding normal subgroups (and subgroups which are *not*.)

3. In part (c) the brackets denote "the subgroup generated by. . . ." So part (c) is very different from part (b). The answer to part (b) requires describing an infinite set; the answer to part (c) is a finite list.

None of these problems should be difficult.